CHAPTER 14

The Mechanical Energy Balance

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Your objectives in studying this chapter are to be able to

1. Explain what a reversible process is
2. Identify a process as reversible or irreversible given a description of the process
3. Define efficiency and apply the concept to calculate the work for an irreversible process
4. Write down each of the terms in the steady-state mechanical energy balance for an open system, and understand the conditions under which the equation can be applied
5. Apply the mechanical energy balance when appropriate to problems so that you can predict pressure drops, velocities, friction losses, and pump sizes

The ideal process is a hypothetical process that rarely occurs in practice. Why bother with it then? The reason is that calculations for energy changes can be made for an ideal process, and then an empirical *efficiency* used to convert the ideal work or energy change into the actual work or energy change. Understanding how nonideal (real) processes degrade the usefulness of various kinds of energy will help you avoid investing in perpetual motion machines.

Looking Ahead

In this chapter we examine certain types of ideal processes called reversible processes. Then we go on to discuss various kinds of efficiencies. Finally, we

describe and apply to practical problems a type of energy balance called the mechanical energy balance.

14.1 Ideal Reversible Processes

If you can keep your head when all about you are losing theirs, maybe you just don't fully understand the situation.

Sign in a Pentagon office

The critical issue in designing and operating processes is not the conservation of energy, for example, applying the general energy balance. Energy in the system plus the surroundings (the **universe**) is *always* conserved. What is of concern is how energy can be converted from one form to another, and how efficiently the conversion can be made to take place. To characterize the interactions of energy within a system, and between the system and its surroundings, as the system moves from one state of equilibrium to another, scientists posit two types of processes:

1. **Reversible:** an idealized process (primarily hypothetical) in which changes occur because of an infinitesimal (differential) imbalance of temperature, pressure, and so on
2. **Irreversible:** a process that is not reversible because it occurs with finite differences of temperature, pressure, and so on—most real processes

As you might infer from the name, a reversible process is one in which the states of the system plus the surroundings can be reversed by a differential change in the driving potential (dT, dp, etc.). What we do is to characterize a process in words as "a controlled, very slow expansion," "frictionless," "elastic spring," and so on, so that it can be treated as reversible in your calculations. Examine Figure 14.1.

Example 9.2 illustrated the calculation of reversible work. Note that the piston in Figure 14.1 moves vertically so that the potential energy of the center of mass of the gas changes slightly on expansion.

Irreversible processes involve dissipative effects and nonequilibrium transfers of energy. Both types of processes increase molecular disorder. Examples are the mixing of dissimilar substances, electrical resistance, sudden changes in phase, mass transport under finite concentration difference, pipe flow, and so on. In Figure 14.1b, what happens to the work being done by the gas on the piston during the expansion of the gas if the process is irreversible? Some of the work causes the piston to rise and thus stores the

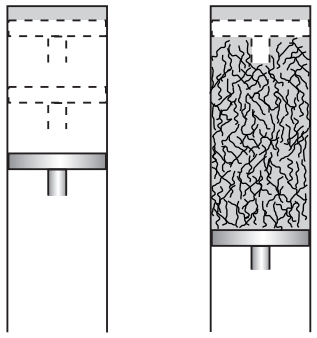


Figure 14.1 Comparison of a hypothetical reversible process shown in Figure 14.1a with an irreversible process shown in Figure 14.1b: (a) A very slow expansion of gas without friction between the piston and the chamber wall; (b) a fast expansion of a gas with friction between the piston and the wall and turbulence

energy in a device that later on can itself do work. But some of the work is converted to thermal energy (U) in the piston, the cylinder, and the gas, energy that cannot be completely recovered as work. Thus, if the piston were returned to its original state in Figure 14.1b by doing work to compress the gas, you would find that the surroundings had to supply some extra work to complete the return to the original state.

Example 14.1 Calculation of the Work Done during Evaporation of a Liquid

How much work is done by 1 L of saturated liquid water when it evaporates from an open vessel into the atmosphere where the pressure is 100 kPa?

Solution

Example 14.1 illustrates one real process that is essentially reversible.

Steps 1–4

The system is the water. The process is an open system. Does the water do work in evaporating? Certainly! It does work against the atmosphere by pushing it back. Furthermore, the process diagrammed in Figure E14.1, is a reversible one because the evaporation takes place at constant temperature (372.78 K) and pressure, and presumably the conditions in the atmosphere immediately above the open portion of the vessel are in equilibrium with the water surface. The atmospheric pressure is 100 kPa. The basis will be 1 L of liquid water that vaporizes to a gas. The specific volume of the water vapor is 1.694 $m^3/kg\ H_2O$.

Step 5

Basis: 1 L of water (liquid)

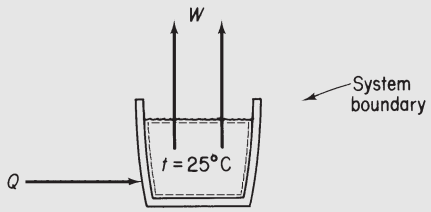


Figure E14.1

Steps 6 and 7

The unknown quantity is the work done by the liquid water pushing against the atmosphere. The general energy balance

$$\Delta E = Q + W - \Delta[(\hat{H} + \hat{KE} + \hat{PE})m]$$

will not be useful in solving this problem because Q is unknown. Even though the system is actually an open one, imagine that an expandable bag is placed over the open face of the vessel so that the system consisting of the water becomes a closed system. Because of the reversible assumption established for this problem, the work for a closed system is

$$W = -\int_{V_1}^{V_2} p dV = -p\Delta V$$

which represents the reversible work done by the water filling the hypothetical bag in pushing back the atmosphere.

Steps 8 and 9

The specific volume of the liquid water from the SI steam tables is 0.001043 m^3/kg. The final volume of the water vapor is (the initial volume of H_2O in the bag is zero and we will assume that there is enough water in the vessel so that the loss of liquid water to vapor is negligible)

$$\frac{1 \text{ L liquid}}{}\left|\frac{1 \text{ m}^3}{1000 \text{ L}}\right|\frac{1 \text{ kg}}{0.001043 \text{ m}^3 \text{ liquid}}\left|\frac{1.694 \text{ m}^3 \text{ vapor}}{\text{kg}}\right. = 1.624 \text{ m}^3 \text{ vapor}$$

$$W = -\frac{100 \times 10^3 \text{ Pa}}{}\left|\frac{1 \text{ (N)/(m}^2\text{)}}{1 \text{ Pa}}\right|\frac{1 \text{ J}}{1 \text{ (N)(m)}}\left|\frac{1.624 \text{ m}^3}{}\right. = -1.624 \times 10^5 \text{ J}$$

Example 14.2 Calculation of the Work in a Batch Process

One kilogram mole of N_2 gas is in a horizontal cylinder at 1000 kPa and 20°C. A 6 cm^2 piston of 2 kg mass seals the cylinder and is fixed by a pin. The pin is released, and the N_2 volume is doubled, at which time the piston is stopped again. What is the work done by the gas in this process?

Solution

Steps 1–4

Draw a picture. See Figure E14.2. From the ideal gas law you can compute the specific volume of the gas at the initial state:

$$\hat{V}_1 = \frac{RT}{p} = \frac{8.31\ (\text{kPa})(\text{m}^3)}{(\text{K})(\text{kg mol})}\left|\frac{293\ \text{K}}{1000\ \text{kPa}}\right. = 2.435\ \text{m}^3/\text{kg mol}$$

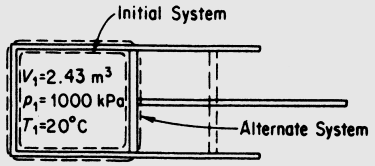


Figure E14.2

a. Is the process a flow or a nonflow process? Since no material leaves or enters the cylinder, let us analyze the process as a nonflow system.

b. This process is definitely irreversible because of friction between the piston and the walls of the cylinder, turbulence in the gas, the large pressure drops, and so on.

Choose the gas to be the system.

Step 5

We can use as a basis 2.435m^3 of N_2 at 1000 kPa and 20°C.

Steps 6 and 7

We want to calculate the work done by the gas on the piston and the cylinder. As in Example 14.1, the work cannot be calculated from the general energy balance because Q is unknown. Furthermore, because the process is irreversible, you cannot calculate the work done on the piston by $-\int p\,dV$.

Let us change our system to include the gas, the piston, and the cylinder. With this choice, the system expansion is still irreversible, but the pressure in the surroundings can be assumed to be atmospheric pressure and is

constant. The work done in pushing back the atmosphere probably is done almost reversibly from the viewpoint of the surroundings, as assumed in Example 14.1, and can be closely estimated by calculating the work done by the alternate system on the surroundings.

$$W = -\int_{V_1}^{V_2} pdV = -p\Delta V$$

ΔV of the gas is $2V_1 - V_1 = V_1 = 2.435\ \text{m}^3$, so that the volume change of the surroundings is $-2.435\ \text{m}^3$. Then the work done on the surroundings is

Basis: 2.435 m^3 at 101.3 kPa and 20°C (assumed)

$$W = -\frac{101.3 \times 10^3\ \text{Pa}}{}\left|\frac{-2.435\ \text{m}^3}{}\right|\left|\frac{1(\text{N})(\text{m}^{-2})}{1\ \text{Pa}}\right|\frac{1\ \text{J}}{1(\text{N})(\text{m})} = 247\ \text{kJ}$$

From this we know that the work done by the alternate system is

$$W_{\text{system}} = -W_{\text{surroundings}} = -247\ \text{kJ}$$

This answer still does not answer the original question of what work was done by the gas, because the revised system now includes both the piston and the cylinder as well as the gas. You should also note that if the surroundings were a vacuum instead of air, then *no work* would be done by the system consisting of the piston plus the cylinder plus the gas, although the gas itself probably still would do some work on the piston.

How would the above calculation be changed if the piston/cylinder/gas equipment were vertical rather than horizontal? Hint: Think about potential energy.

Self-Assessment Test

Questions

1. The following statement appeared in a text:

"The work performed for any gas process is represented by the area under a pressure-volume diagram and may be found by the following expression."

$$W = \int pdV$$

Comment on this statement.

2. An ideal gas produces 475 J of work in a reversible expansion in an isothermal process; $p_1 = 1$ atm and $p_2 = 0.1$ atm. You are asked to calculate Q, ΔH, and ΔE for this expansion. Can you do so?

3. Can you apply $\int pdV$ to calculate work in a batch reversible process for which the system is a liquid? A solid?

4. Which process will yield more work: (a) expansion of a gas confined by a piston against constant pressure, or (b) reversible expansion of a gas confined by a piston?
5. Define a reversible process.

Problems

1. Find Q, W, ΔE, and ΔH for the reversible compression of 3 mol of an ideal gas from a volume of 100 dm^3 to 2.4 dm^3 at a constant temperature of 300 K.
2. You let 1 mole of an ideal gas expand against an external pressure of 1.12 atm. The initial volume of the gas is 23.17 dm^3, and the final volume is 35.22 dm^3. What is the work done on the system in this process? (Assume $P_{sys} = P_{ext}$ at the end of the process, and that T is constant.) Then you carry out by some means a reversible isothermal expansion of the same ideal gas. Will the work be larger or smaller in the second case? Calculate it.

Thought Problem

Look at Figure SAT14.1TP1. This apparatus appeared in a magazine with the statement that once the funnel was filled it ran continuously for 12 hr before stopping. Do you believe it?

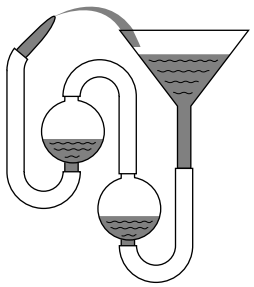


Figure SAT14.1TP1

Discussion Problem

Discuss the feasibility of the following novel proposal as a way to extract energy from river flow that would not require large dams. See Figure SAT14.1DP1. The apparatus consists of a pair of tall chambers connected at the top by a tube containing an air turbine the diameter of a dinner plate. As the water flows through the system, one chamber fills, pushing air past one side of the turbine; the other side empties, sucking air from the tube.

Whereas a standard turbine uses a conventional fan blade that spins according to the direction of the water, the counter-rotating turbine in the proposed system uses a blade shaped like an airplane wing that creates lift to spin the turbine when the airflow rushes over its front edge. The blades are placed around the turbine in paddle-wheel

fashion. As the first water chamber fills, air whooshes through the tube and hits the front of the paddle wheel, lifting each blade upward as it spins past the airflow. As the first chamber empties and the other fills, air races in the opposite direction through the tube and over the blades at the rear of the paddle wheel. The blades, having spun halfway around the wheel, are now upside down, so the "lift" pulls them downward. Thus, even though the airflow reverses with each cycle, the turbine rotates in the same direction, generating a constant flow of electricity.

Simple gates and counterbalances at the bottom coordinate the operation of the unit as the chambers alternately fill and empty. At the top of the cycle, water pressure inside the filled chamber exceeds the pressure exerted by a counterweight on the outflow door at the bottom of the chamber, popping it open. The same motion closes the outflow gate in the opposite chamber, thanks to a single shaft that controls the two gates

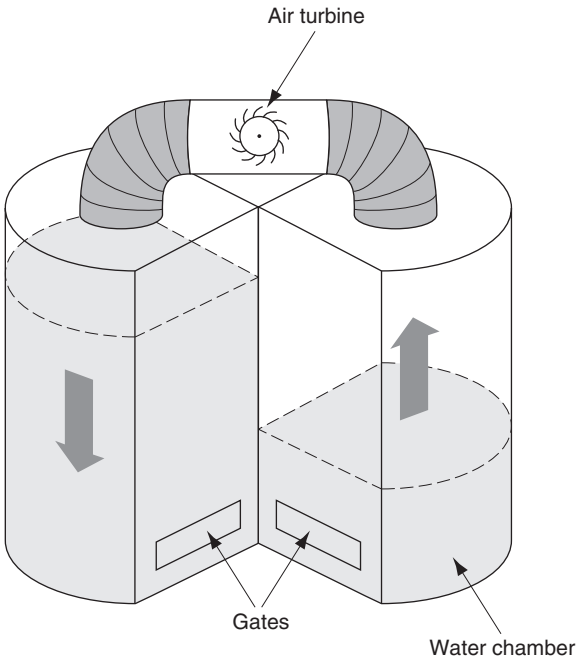


Figure SAT14.1DP1

14.2 Efficiency

If you look in the dictionary for the meaning of the word **efficiency,** you find "the ratio of the work done by an organism or machine to the amount of food or fuel consumed and to the energy expended." In engineering you

will encounter a variety of more precise definitions of efficiency, some of which are listed in Table 14.1. Numerous other specialized quantities termed efficiencies exist for hydroelectric plants; other power plants; automobile engines, transmissions, and drive trains; furnaces; combustion processes; and so on. Table 14.2 lists some typical values of efficiencies of common equipment.

Example 14.3 Efficiency of Power Generation by a Hydroelectric Plant

A small hydroelectric plant puts out 20 MW when the level of water above the generators is 25 m and the water flow rate through the generators is 100 m^3/s. What is the overall efficiency of the plant?

Solution

The mass flow rate of the water (density 1000 kg/m^3) is

$$\frac{100\ \text{m}^3}{\text{s}}\left|\frac{1000\ \text{kg}}{\text{m}^3}\right. = 10^5\ \text{kg/s}$$

The potential energy change of the water per second is

$$\Delta\dot{PE} = \dot{m}gh = \frac{10^5\ \text{kg}}{\text{s}}\left|\frac{9.807\ \text{m}}{\text{s}^2}\right|\frac{25\ \text{m}}{}\left|\frac{(\text{s}^2)(\text{J})}{1(\text{kg})(\text{m})}\right|\frac{(\text{s})(\text{W})}{1\ \text{J}} = 2.45 \times 10^7\ \text{W}$$

$$\eta = \frac{20 \times 10^6\ \text{W}}{2.45 \times 10^7\ \text{W}} = 0.82$$

For a plant with multiple units you can compute the overall efficiency by multiplying together the efficiencies of the individual components. As an example, assume that the conversion of fuel in a power plant yields 88 kJ in the steam product per 100 kJ of available energy from the coal being burned. Also assume that the conversion of the energy in the steam to mechanical energy is 43% efficient, and the conversion of the mechanical to electrical energy is 97% efficient. The overall efficiency is (0.88)(0.43)(0.97) = 0.37, meaning that two-thirds of the initial energy is dissipated as heat to the environment. These definitions provide a good way to compare process performance for energy conservation (but not the only way).

Table 14.1 Various Process Efficiencies

General efficiency

$$\eta_1 = \frac{\text{useful energy out}}{\text{energy in}}$$

Mechanical efficiency

$$\eta_2 = \frac{\text{actual work output from the process}}{\text{work output for a reversible process}}$$

$$\eta_3 = \frac{\text{work input for a reversible process}}{\text{actual work input for the process}}$$

Heat engine efficiency

$$\eta_4 = \frac{\text{actual work output from the process}}{\text{heat input from a source}}$$

Refrigeration cycle efficiency

$$\text{Coefficient of performance: COP} = \frac{\text{heat removed from process}}{\text{work input to the process}}$$

Table 14.2 Efficiencies of Energy Conversion

	TO				
FROM	**Chemical**	**Electrical**	**Mechanical**	**Radiant**	**Thermal**
Chemical		Dry cell battery (90) Storage battery (70) Fuel cell (60) Gas turbine (30)	Automobile engine (37) Aircraft engine (37)		Boilers (88) Home gas furnace (85) Home oil furnace (65) Steam power plant (40)
Electrical	Storage battery (70)			Incandescent lamp (5)	
Mechanical		Electric generator (98)			
Radiant		Solar cell (10)			
Thermal		Thermocouple (8)			

Example 14.4 Calculation of Plant Efficiency

An analysis of energy usage takes place in all plants. This example uses data from M. Fehr, "An Auditor's View of Furnace Efficiency," in *Hydrocarbon Processing* (November, 1988), p. 93. Further details about the process equipment and energy auditing can be found in the article.

Figure E14.4 illustrates a gas-fired boiler. The data calculated from measurements on the heater were (all in kilojoules per cubic meter at S.C. of fuel gas):

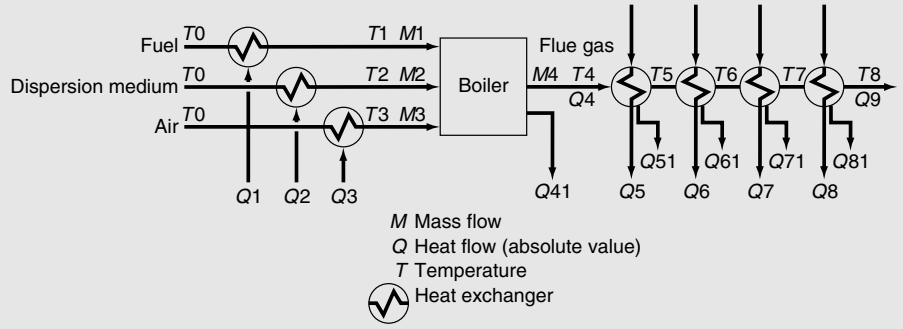


Figure E14.4

LHV of fuel:	36,654	$Q4$:	32,114
$Q1$:	16	$Q41$:	6988
$Q2$:	0	$Q8$:	1948
$Q3$:	2432	$Q9$:	2643

Calculations from measurements gave values for sums of the other Qs of

$$Q5 + Q6 + Q7 = 9092$$

$$Q5 + Q6 + Q7 + Q51 + Q61 + Q71 = 27{,}119$$

$$Q8 + Q81 = 2352$$

From these heat transfer values, several efficiencies could be determined to evaluate plant performance:

a. Gross efficiency

$$\frac{\text{LHV} + Q1 + Q2 + Q3 - Q9}{\text{LHV}} = \frac{36{,}654 + 16 + 0 + 2432 - 2643}{36{,}654} = 0.995$$

b. Thermal efficiency

$$\frac{Q5 + Q6 + Q7 + Q8}{\text{LHV} + Q1 + Q2 + Q3} = \frac{9092 + 1948}{36{,}654 + 16 + 0 + 2432} = 0.282$$

c. Combustion efficiency

$$\frac{Q4}{Q4 + Q41} = \frac{32{,}114}{32{,}114 + 6988} = 0.821$$

Self-Assessment Test

Questions

1. A number of automobile company executives together with the White House announced a program aimed at developing a new generation of automobiles with three times the fuel efficiency of existing cars. They said they were seeking "nothing less than a major, even radical breakthrough in automotive technology that can dramatically increase fuel efficiency, improve safety, and reduce emissions—while maintaining the affordability, performance, and utility of current comparable vehicles."

   Is such a program possible?
2. Which is more efficient in heating water: (a) an electric coffeepot, (b) a microwave oven, or (c) a pot on an electric stove?

Problems

1. A standard heating system in a standard house has the following characteristics: A gas furnace delivers energy to the living space at an average rate of 5.8 kW on a typical winter day. One-fourth of the energy that comes from the fuel input to the furnace is lost through incomplete combustion and the escape of heat to the outdoors via the flue. Of the remaining energy, 12% escapes through duct walls and doesn't reach the living space. Calculate the efficiency of the use of the gaseous fuel.
2. Cogeneration plants produce both electricity and steam. Figure SAT14.2P2 shows such a plant. What will be the percent fuel savings to get the electricity and steam from using the cogeneration plant in contrast to using two separate plants, one to produce 0.25 MJ of electric energy and one to produce 0.45 MJ of thermal energy in the steam?

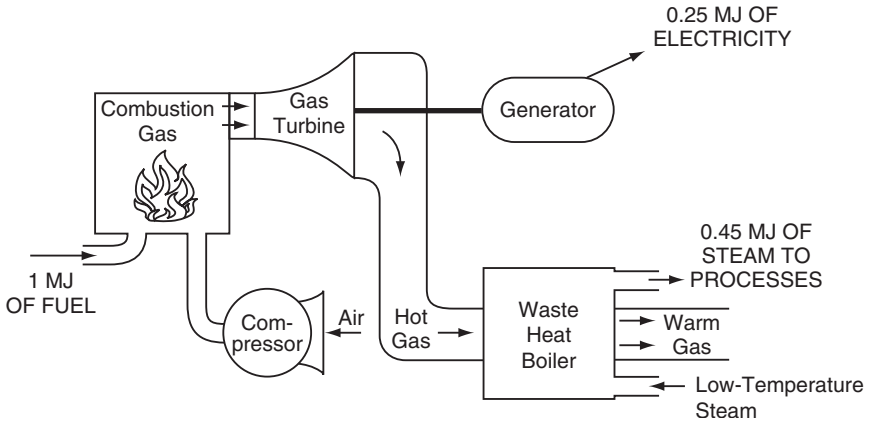


Figure SAT14.2P2

3. Calculate the reversible work required to compress 5 ft^3 of an ideal gas initially at 100°F from 1 to 10 atm in an adiabatic cylinder. Such a gas has an equation of state $pV^{1.40}$ = constant. Then calculate the actual work required if the efficiency of the process is 80%.

Thought Problems

1. A news announcement in a professional journal described a 20 hp Stirling engine that was going to be connected to and drive a 68 kW generator. Would you buy one of these at $4500?
2. An author compared the effectiveness of gasoline-powered automobiles with electric-powered automobiles as in the following table, and concluded that electric-powered vehicles overall were more efficient. Is the comparison valid?

Efficiencies	Gasoline (bbl of oil)	Electric (bbl of oil)
KWh equivalent	1700	1700
Refinery efficiency	74% to gasoline	89% to heavy oil
Distribution efficiency	95%	
Power generation efficiency		40%
Power distribution efficiency		91%
Battery efficiency		70%
Motor, drive train efficiency	14.7%	80%
KWh available	175.7	308.8
Vehicle weight	2400 lb	2800 lb
Road load: 50 mph, level	8.4 kW	10 kW
Distance traveled	1045 miles	1545 miles

3. A clipping from the *Wall Street Journal* reads:

   Technical Disputes

   Furnace efficiency is a comparison of the energy that goes into a furnace with the usable heat that comes out. Because oil furnaces use blowers to burn the fuel more efficiently and because oil produces less water vapor than gas, less heat is vented up the chimney, the Oil Jobbers Council says. The American Gas Association complains that such calculations ignore variations in oil quality that hurt efficiency. And it contends that oil furnaces are apt to "lose efficiency over time," while it says gas furnaces don't.

   Which organization is correct, or is neither correct?

14.3 The Mechanical Energy Balance

You can never enter the same river twice.

Ancient Chinese proverb

Examine Figure 14.2. What size pump should be selected to remove biowaste from the collection tank? What pressure drop occurs between the collection tank drain and the pipeline to the treatment system?

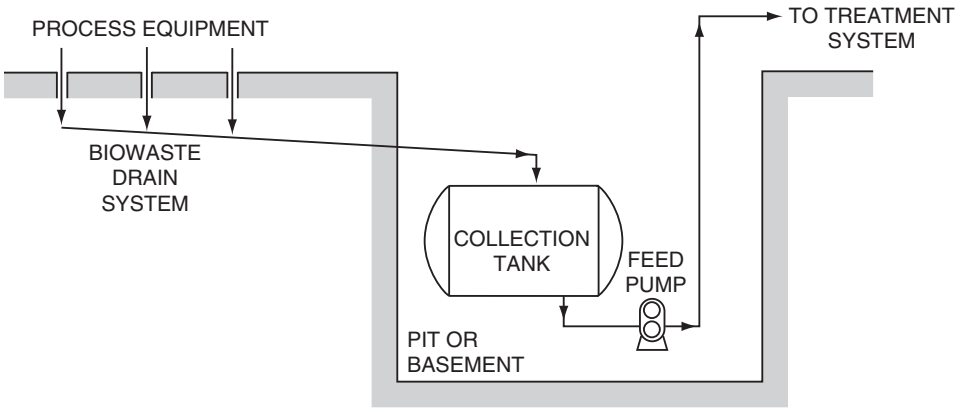


Figure 14.2 Biowaste collection system

Such questions can be answered by using what is called the mechanical energy balance rather than the general energy balance.

In some processes, such as distillation columns or reactors, in which heat transfer and enthalpy changes are *the* important energy components in the energy balance, work, potential energy, and kinetic energy terms can be deemed to be zero or quite minor. However, in other processes, such as the compression of gases and pumping of liquids, work and the mechanical forms of energy are *the* important factors. For these processes, an energy balance, including solely the mechanical forms of energy, becomes a useful tool.

Two categories of energy of different "quality" can be distinguished:

a. The so-called *mechanical* forms of energy, such as kinetic energy, potential energy, and work, which are *completely* convertible by an *ideal* (reversible) engine from one form to another within the class

b. Other forms of energy, such as internal energy and heat, which are not so freely convertible

Of course, as we explained in Section 14.1, in any real process with friction, viscous effects, mixing of components, and other dissipative phenomena taking place that prevent the complete conversion of one form of mechanical energy to another, allowances will have to be made in making a balance on mechanical energy for these "losses" in quality.

A balance on mechanical energy can be written on a microscopic basis for an elemental volume by taking the scalar product of the local velocity and the equation of motion.[1] After integration over the entire volume of

[1]J. C. Slattery, *Momentum, Energy, and Mass Transfer,* 2nd ed., Krieger, New York (1981).

the system, the ***steady-state mechanical energy balance*** **for a system with mass interchange with the surroundings** becomes, on a per unit mass basis:

$$\Delta(\widehat{KE} + \widehat{PE}) + \int_{p_1}^{p_2} \hat{V}\, dp - \hat{W} + \hat{E}_v = 0 \tag{14.1}$$

where $\widehat{KE}$ and $\widehat{PE}$ are associated with the mass flow in and out of the system, and E_v represents the loss of mechanical energy, that is, the irreversible conversion by the *flowing fluid* of mechanical energy into nonmechanical forms of energy, a term that must in each individual process be evaluated by experiment (or, as occurs in practice, by use of already existing experimental results for a similar process). Equation (14.1) is called the **Bernoulli equation** for a reversible process in which $\hat{E}_v = 0$. If the fluid is incompressible, and $W = 0$

$$\frac{\Delta p}{\rho} + \frac{\Delta v^2}{2} + g\Delta h = 0 \tag{14.2}$$

The mechanical energy balance is best applied in fluid-flow calculations when the kinetic and potential energy terms and the work are of importance, and the friction losses can be evaluated from handbooks with the aid of *friction factors* or *orifice coefficients.*

Let us now look at two typical applications of the steady-state mechanical energy balance.

Example 14.5 Comparison of the Reversible Work for a Batch Process with That of a Flow Process Operating under the Same Conditions

Five cubic feet of an ideal gas at 100°F are to be compressed adiabatically from 1 atm to 10 atm. The equation of state to use for the gas is $pV^{1.40} = \text{constant}$. Two options exist for the compression:

a. Compression in a horizontal cylinder with a piston
b. Compression in a rotary compressor

Calculate the reversible work required for the two alternate processes.

Solution

Process a
You can represent this process as a closed system similar to Figure 14.1, but with the cylinder horizontal. The general energy balance reduces to

$$\Delta E = \Delta U = W$$

If you had a value for C_v you could integrate $C_v dT$ and get W, but no value is given for C_v (or C_p). However, you can calculate W by integrating pdV for the reversible work, as shown in Examples 14.1 and 14.2.

Steps 1–4

You know the value of V_1 and have to calculate the value of V_2. The system is the gas.

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/1.40} = 5\left(\frac{1}{10}\right)^{1/1.40} = 0.965 \text{ ft}^3$$

Step 5

Basis: 5 ft^3 of gas at 100°F and 1 atm

Steps 6–9

$$W_{\text{rev}} = -\int_{V_1=5}^{V_2=0.965} pdV = -\int_{V_1}^{V_2} p_1\left(\frac{V_1}{V}\right)^{1.40} dV = -p_1 V_1^{1.4}\int_{V_1}^{V_2} V^{-1.40}\, dV$$

$$= -\frac{p_1 V_1^{1.40}}{1 - 1.40}(V_2^{-0.40} - V_1^{-0.40}) = -\frac{P_2V_2 - p_1V_1}{1 - 1.40}$$

$$= -\frac{[(10)(0.965) - (1)(5)](\text{ft}^3)(\text{atm})}{-0.40}\left|\frac{1.987 \text{ Btu}}{0.7302(\text{ft}^3)(\text{atm})}\right.$$

= 31.6 Btu (the positive sign indicates work done on the system).

Process b

You can represent this process as an open, steady-state system and apply the mechanical energy balance to solve for the reversible work:

$$\Delta(\hat{KE} + \hat{PE}) + \int_{p_1}^{p_2} \hat{V}\, dp - \hat{W} + \hat{E}_v = 0$$

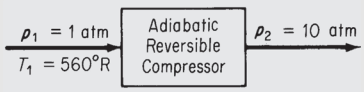


Figure E14.5

Figure E14.5 represents the process. The $\hat{KE}$ and $\hat{PE}$ terms are assumed to be zero, and $\hat{E}_v = 0$ because of the assumption of reversibility. Thus

$$\hat{W} = \int_{p_1}^{p_2} \hat{V} dp$$

(*Continues*)

Example 14.5 Comparison of the Reversible Work for a Batch Process with That of a Flow Process Operating under the Same Conditions (*Continued*)

Step 1–4

Figure E14.5 designates the system and data. The moles of gas are

$$n_1 = \frac{p_1 V_1}{RT_1} = \frac{1 \text{ atm}}{}\left|\frac{5 \text{ ft}^3}{}\right|\frac{}{560°\text{R}}\left|\frac{1 \text{ (lb mol)(°R)}}{0.7302 \text{ (ft}^3\text{)(atm)}}\right. = 0.0122 \text{ lb mol}$$

Step 5

$$\text{Basis} = 0.0122 \text{ lb mol}$$

Steps 3, 6, 7, 8, and 9

$$\hat{W}_{rev} = \int_{p_1}^{p_2} \hat{V} dp = \int_{p_1}^{p_2} \hat{V}_1 \left(\frac{p_1}{p}\right)^{1/1.4} dp$$

$$= \hat{V}_1 p_1^{0.714}[3.50(p_2^{0.286} - p_1^{0.286})]$$

$$\hat{W}_{rev} = n_1 \hat{W}_{rev} = n_1 \left(\frac{V_1}{n_1}\right) p_1^{0.714}[3.50(p_2^{0.286} - p_1^{0.286})]$$

$$= (5)(1)^{0.714}[(3.50)(10)^{0.286} - 1^{0.286}](\text{ft}^3)(\text{atm})\left(\frac{1.987 \text{ Btu}}{0.7302(\text{ft}^3)(\text{atm})}\right)$$

$$= 44.3 \text{ Btu}$$

Example 14.6 Application of the Mechanical Energy Balance to the Pumping of Water

Calculate the work per minute required to pump 1 lb of water per minute from 100 psia and 80°F to 1000 psia and 100°F. The exit stream is 10 ft above the entrance stream.

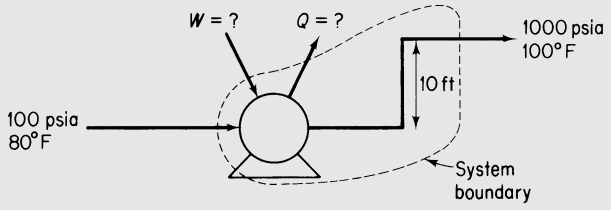


Figure E14.6

Solution

This problem is typical of many fluid flow problems.

Steps 1–4

The system shown in Figure E14.6 is a steady-state process and the data have been placed in the figure.

Steps 4, 6, and 7

The general mechanical energy balance is

$$\Delta(\hat{KE} + \hat{PE}) + \int_{p_1}^{p_2} \hat{V} dp - \hat{W} + \hat{E}_v = 0 \tag{a}$$

We assume that $\Delta\hat{KE}$ is insignificant, that (preliminarily) the process is reversible so that $E_v = 0$, and that the pump is 100% efficient. (Subsequently, we will consider what to do if the process is not reversible.) Equation (a) reduces to

$$\hat{W} = \int_{p_1}^{p_2} \hat{V}\, dp + \Delta\hat{PE} \tag{b}$$

Step 5

Basis: 1 min of operation = 1 lb H_2O

Steps 6 and 8

From the steam tables, the specific volume of liquid water is 0.01607 ft^3/lb_m at 80°F and 0.01613 ft^3/lb_m at 100°F. For all practical purposes the water is incompressible, and the specific volume can be taken to be 0.0161 ft^3/lb_m. We have only one unknown in Equation (b): $\hat{W}$.

Step 9

$$\Delta\hat{PE} = \frac{10 \text{ ft}}{} \left| \frac{32.2 \text{ ft}}{\text{sec}^2} \right| \left| \frac{}{\frac{32.2(\text{ft})(\text{lb}_m)}{(\text{lb}_f)(\text{sec}^2)}} \right| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} = 0.0129 \text{ Btu}$$

$$\int_{100}^{1000} 0.0161\, dp = \frac{0.0161 \text{ ft}^3}{\text{lb}_m} \left| \frac{(1000 - 100)\text{lb}_f}{\text{in.}^2} \right| \left| \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 \right| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} = 2.68 \text{ Btu}$$

$$\hat{W} = 2.68 + 0.0129 = 2.69 \frac{\text{Btu}}{\text{lb}_m}$$

(*Continues*)

Example 14.6 Application of the Mechanical Energy Balance to the Pumping of Water (*Continued*)

About the same value can be calculated using the general energy balance (if $\hat{Q} = \widehat{KE} = 0$) and the steam tables, because the enthalpy change for a reversible process for 1 lb of liquid water going from 100 psia and 100°F to 1000 psia is 2.70 Btu. Make the computation yourself. However, usually the enthalpy data for liquids other than water are missing, or not of sufficient accuracy to be valid, which forces you to turn to the mechanical energy balance.

You might now well inquire for the purpose of purchasing a pump-motor as to what the work would be for a real process instead of the fictitious reversible process assumed above. First, you would need to know the efficiency of the combined pump and motor so that the actual kilowatt input from the surroundings (the electrical connection) to the system would be known. Second, the friction lost in the pipe, valves, and fittings must be estimated so that the term $\hat{E}_v$ could be reintroduced into Equation (a). Suppose, for the purposes of illustration, that $\hat{E}_v$ was estimated to be, from an appropriate handbook, 320 (ft)$(\text{lb}_\text{f})/\text{lb}_\text{m}$ and the pump-motor efficiency was 60% (based on 100% efficiency for a reversible pump-motor). Then,

$$\hat{E}_v = \frac{320(\text{ft})(\text{lb}_\text{f})}{1\ \text{lb}_\text{m}} \left| \frac{1\ \text{Btu}}{778(\text{ft})(\text{lb}_\text{f})} \right. = 0.41\ \text{Btu}/\text{lb}_\text{m}$$

$$\hat{W} = 2.68 + 0.013 + 0.41 = 3.10\ \text{Btu}/\text{lb}_\text{m}$$

Remember that the positive sign indicates that work is done on the system. The actual pump-motor must have the capacity

$$\frac{3.10\ \text{Btu}}{\text{lb}} \left| \frac{1\ \text{lb}}{\text{min}} \right| \frac{1}{0.60} \left| \frac{1\ \text{min}}{60\ \text{sec}} \right| \frac{1.415\ \text{hp}}{1\ \text{Btu/sec}} = 0.122\ \text{hp}$$

In Equation (14.2), if each term is divided by g, the units of the equation are those of Δh and are known as "heads" of fluid. Thus, the 10 ft of water in Example 14.6 would be called a head of 10 feet of water.

A head of water is not a head full of water.

Anonymous

Self-Assessment Test

Questions

1. What assumptions are built into Bernoulli's equation? Give two examples of a system where this equation would be useful.
2. Why can the terms in the Bernoulli equation be called heads?

3. Is the Bernoulli equation used more or less than the mechanical energy balance?
4. A textbook stated that "fluids can only flow from high pressure to low like electric charge can only flow from high potential to low." Is this true?
5. Explain why the flow out of a garden hose 10 m in length attached to the city water supply is more than the flow from a 100 m garden hose.

Problems

1. Water is pumped from a very large storage reservoir, as shown in Figure SAT14.3P1, at the rate of 2000 gal/min. Determine the minimum power (i.e., that for a reversible process) required by the pump in horsepower.

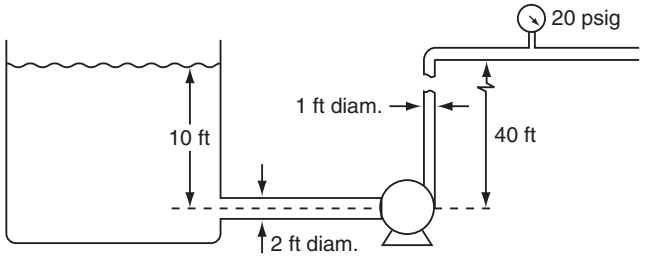


Figure SAT14.3P1

2. Use the mechanical energy balance to explain why the flow of water down a vertical pipe that is full of water differs from the flow of water down a waterfall.
3. Fluid with a specific gravity of 1.15 drains from the bottom of a large open tank into a bucket through a 5 cm (inner diameter) pipe that terminates 4.5 m below the surface of the fluid in the tank. Calculate the velocity of discharge of the fluid, ignoring frictional effects.

Thought Problems

1. Examine Figure SAT14.3TP1. Is it possible by blowing in the stem of the funnel that the Ping-Pong ball will rise toward the stem rather than fall down? Why?

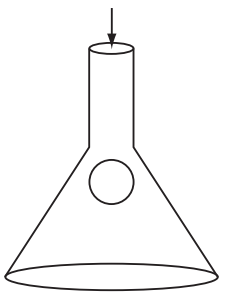


Figure SAT14.3TP1

2. Examine Figure SAT14.3TP2. List the figures that correctly represent the initial flow of water from the respective (full) columns.

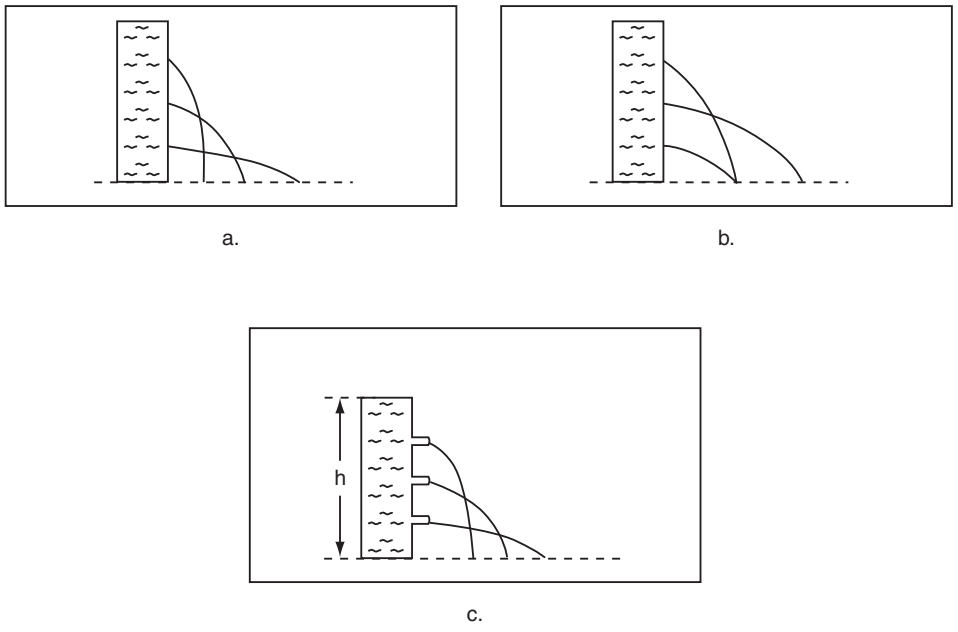


Figure SAT14.3TP2

Looking Back

In this chapter we explained the concepts of reversible (ideal) and irreversible processes. Process efficiency is a measure of how close to ideal a process is. We also added the mechanical energy balance to the list of tools you can use in analyzing processes, because it accounts for the conversions between mechanical forms of energy (*KE, PE, W*).

Glossary

Bernoulli equation The steady-state mechanical energy balance for a reversible process with no work.

Coefficient of performance (COP) The heat removed from a process divided by the work input to the process.

Efficiency The ratio of the work done by an organism or machine to the amount of food or fuel consumed and/or to the energy expanded.

Efficiency, general The energy output divided by the energy input.

Heat engine efficiency The actual work output from a process divided by the heat input from a source.

Irreversible process A process that is not reversible.

Mechanical efficiency (a) The actual work output from a process divided by the work output if the process were reversible; or (b) the work input for a reversible process divided by the actual work input for the process.

Mechanical energy Type of energy that can be completely converted from one form to another.

Reversible process An idealized process in which changes occur under a differential imbalance in temperature, pressure, etc.

Universe The system plus its surroundings.

Supplementary References

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Glasstone, S. *Energy Deskbook,* Van Nostrand Reinhold, New York (1997).

Li, K. W. *Applied Thermodynamics: Availability Method and Energy Conversion,* Taylor and Francis, London (1995).

Moran, M. J. *Fundamentals of Engineering Thermodynamics,* 4th ed, John Wiley, New York (2000).

Web Sites

http://scholar.chem.nyu.edu/0651/notes/pchem/node11.html

Problems

*__14.1__ A phase change (condensation, melting, etc.) of a pure component is an example of a reversible process because the temperature and pressure remain constant during the change. Calculate the work done in a batch process by butane when 1 kg of saturated liquid butane at 70 kPa vaporizes completely. Can you calculate the work done by the butane from the energy balance alone?

*__14.2__ Calculate the work done when 1 lb mol of water in an open vessel evaporates completely at 212°F. Express your result in British thermal units.

**__14.3__ One kilogram of steam goes through the following reversible process: In its initial state (state 1) it is at 2700 kPa and 540°C. It is then expanded isothermally

to state 2, which is at 700 kPa. Then it is cooled at constant volume to 400 kPa (state 3). Next it is cooled at constant pressure to a volume of 0.4625 m^3/kg (state 4). Then it is compressed adiabatically to 2700 kPa and 425°C (state 5), and finally it is heated at constant pressure back to the original state.

a. Sketch the path of each step in a *p*-*V* diagram.
b. Compute ΔU and ΔH for each step and for the entire process.
c. Compute *Q* and *W* whenever possible for each step of the process.

*14.4 Calculate the work done when 1 lb mol of water evaporates completely at 212°F in the following cases. Express your results in British thermal units per pound mole.

a. A steady-state flow process: water flowing in a pipeline at 1 lb mol/min, neglecting the potential and kinetic energy changes
b. A nonflow process: water contained in a constant-pressure, variable-volume tank

**14.5 A question arose among a group of students concerning a gas expanding by an adiabatic process and at the same time doing no work. One part of the group thinks the process must be isothermal, while the other part insists it cannot be isothermal. They ask you to straighten this matter out.

a. Explain under what circumstances the process will be isothermal.
b. Explain under what circumstances it will not be isothermal.

**14.6 Nitrogen is being compressed by a piston as shown in Figure P14.6.

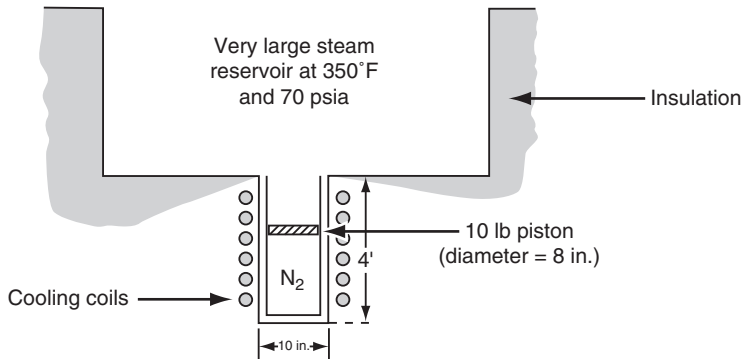


Figure P14.6

One pound mole of nitrogen is initially at 300°F and 14.7 psia in the cylinder. The piston is released and the N_2 compressed by the steam until the final volume of the N_2 is 1/2 the initial volume, at which time the piston is caught and held in place. The barometric pressure is 740 mm. Calculate:

a. The work done by the steam
b. The work done on the N_2
c. The change in internal energy of the N_2

Assume throughout the problem that there is no heat transfer between the piston, cylinder, the steam, and the surroundings.

*14.7 A motor is rated as 30 horsepower (hp). However, the output horsepower of the motor is only 24.6 hp. What is the efficiency of the motor?

*14.8 Estimate the annual fuel cost for a 300 MW coal-fired power plant if the overall efficiency is 40% and the fuel cost is \$1.10/10^6 Btu.

The plant operates for 6000 hr/year.

*14.9 The following measurements have been made on a direct-fired heater using natural gas of the following composition:

CH_4	96.4%
C_2H_6	2.01
C_3H_6	0.6
N_2	0.99

and 10.0% excess air.

Data: Flue gas temperature 450°C

Thermal heat loss 2% of LHV

Calculate the "efficiency" of the heater in percent.

$$\text{Efficiency} = 100 - \left(\frac{\text{Thermal heat loss}}{\text{LHV}} - \frac{\text{Enthalpy of flue gas relative to S.C.}}{\text{LHV}}\right)100$$

*14.10 Three efficiencies are commonly defined for a power plant such as the one shown in Figure P14.10:

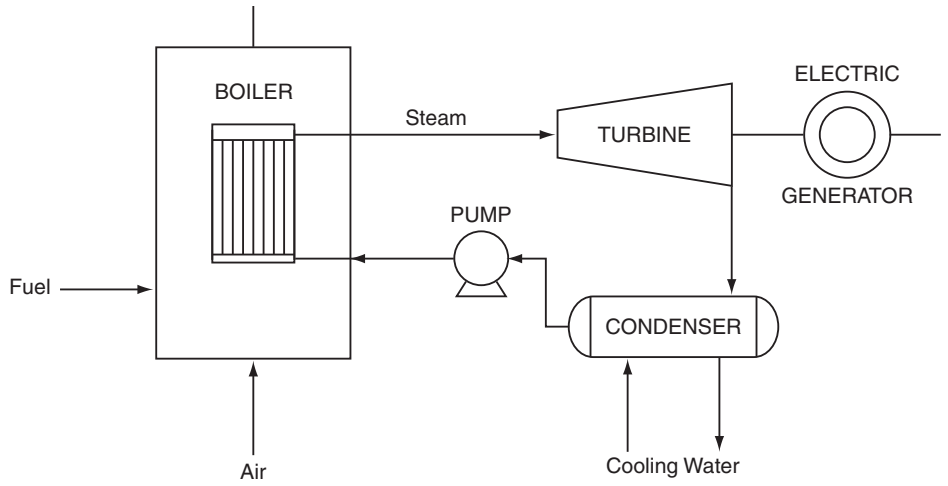


Figure P14.10

$$\text{Boiler efficiency} = \frac{\text{Heat added to the stream}}{\text{Energy input from the fuel}}$$

$$\text{Gross plant efficiency} = \frac{\text{Gross electric output}}{\text{Energy input from the fuel}}$$

$$\text{Steam cycle efficiency} = \frac{\text{Gross electric output}}{\text{Heat added to the steam}}$$

Calculate these efficiencies based on the following data collected for 1 hr of operation of the plant in Figure P14.10:

Gas turbine is 500 MW
Steam flow rate was 640 kg/s
Water entered the boiler as saturated liquid at 150 kPa
Steam left the boiler superheated at 150 kPa and 530°C
The fuel (coal with an LHV of 28,400 kJ/kg) was burned at the rate of 64.6 kg/s

*14.11 A system consists of 5 kg of water vapor at the dew point. The system is compressed isothermally at 400 K, and 400 kJ of work are done on the system by the surroundings. What volume of liquid was present in the system before and after compression?

*14.12 In a processing plant, milk flows from a storage tank maintained at 5°C through a valve to a pasteurizer via an insulated 10-cm-diameter pipe at the rate of 1000 L/min. The upstream pressure is 290 kPa and downstream pressure is 140 kPa. Determine the lost work (E_v) in joules per minute and the temperature change that occurs in the milk as a result of this throttling process. (Milk and water are sufficiently equivalent in properties for you to use those of water.)

*14.13 A power plant is as shown in Figure P14.13. If the pump moves 100 gal/min into the boiler with an overall efficiency of 40%, find the horsepower required for the pump. List all additional assumptions required.

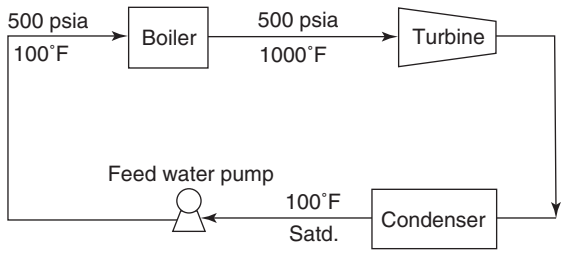


Figure P14.13

*14.14 An office building requires water at two different floors. A large pipe brings the city water supply into the building at the basement level, where a booster pump is located. The water leaving the pump is transported by smaller insulated pipes to the second and fourth floors, where the water is needed. Calculate the *minimum* amount of work per unit time (in horsepower) that the pump must do in order to deliver the necessary water, as indicated in Figure P14.14. (*Minimum* refers to the fact that you should neglect the friction and pump energy losses in your calculations.) The water does not change temperature.

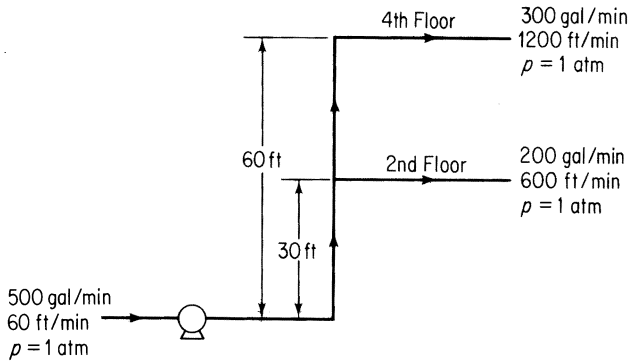


Figure P14.14

*14.15 Water at 20°C is being pumped from a constant-head tank open to the atmosphere to an elevated tank kept at a constant pressure of 1150 kPa in an experiment as shown in Figure P14.15. If water is flowing in the 5.0 cm line at a rate of 0.40 m^3/min, find

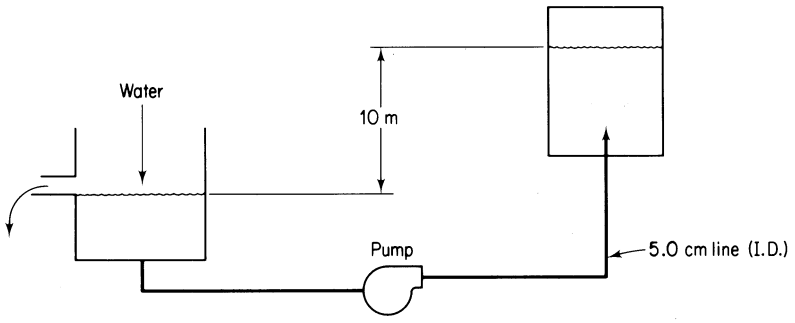


Figure P14.15

a. The rating of the pump in joules per kilogram being pumped
b. The rating of the pump in joules per minute

The pump and motor have an overall efficiency of 70% and the energy loss in the line can be determined to be 60.0 J/kg flowing.

**14.16 An example in a book states that a pump is pumping finish water to a storage tank at 120 ft elevation difference. The pressure gauge reading at the discharge line of the pump is 87.6 psi. The problem was to determine the head loss due to friction.

The solution was:

$$\text{Gauge reading} = \text{elevation pressure} + \text{friction loss}$$

$$\text{Friction head loss} = 87.6 \text{ psi} - 0.433 \text{ psi/ft} \times 120 \text{ ft}$$

$$= 87.6 \text{ psi} - 53.0 \text{ psi}$$

$$= 34.6 \text{ psi}$$

Can the answer be correct for the E_v term in Equation (14.1)?

*14.17 A boiler feed water pump takes suction from a deaerator that operates at 15 psig. To satisfy pump design requirements, the pressure at the suction of the pump must be 24 psig. It is known that $\hat{E}_v$, the irreversible energy loss due to pipe friction, is 4 (ft)(lb_f)/lb_m. The water velocity is 8 ft/s. What is the required height of the saturated water surface above the pump suction, in order to satisfy the 24 psig suction requirement?

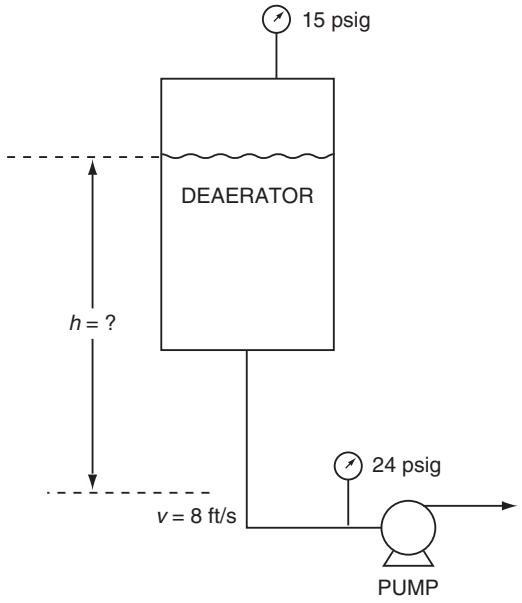


Figure P14.17

*14.18 Solid carbon dioxide (dry ice) has innumerable uses in industry and in research. Because it is easy to manufacture, the competition is severe, and it is necessary to make dry ice very cheaply to be successful in selling it. In a proposed plant to make dry ice, the gaseous CO_2 is compressed isothermally and essentially reversibly from 6 psia and 40°F to a specific volume of 0.05 ft^3/lb_m.

a. What is the final state of the compressed CO_2 per lb?
b. Compute the work of compression.
c. What is the heat removed per pound of CO_2?
d. If the actual efficiency of the compressor (relative to a reversible compressor) is 85% and the electricity to run the compressor motor costs \$0.08/kWh, what is the cost of compression of the solid CO_2 in dollars per pound of dry ice?

*14.19 If a turbine, driven by water flowing from a reservoir 80 m higher than the turbine, delivers 200 kW, what is the flow rate of the water in kilograms per

second? Assume that the friction losses in the system yield an overall efficiency (actual work/ideal reversible work) of 75%. The reservoir is open to the atmosphere, and the exit velocity of the water is 5 m/s at a pressure of 150 kPa from the turbine.

**14.20 A Venturi meter (refer to Figure 2.8) is a machine-cased section of pipe with a narrow throat. The device, in a short cylindrical section, consists of an entrance cone and a diffuser cone which expands to the full pipe diameter. Two openings are installed at the entrance (section 1) and at the throat (section 2). When the water passes through the throat, the velocity increases and the pressure decreases. The decrease of pressure is directly related to the flow. Use the Bernoulli equation and, neglecting friction loss, show that that the volumetric flow rate from the meter is

$$\dot{q} = C_d A_1 \sqrt{2[(\Delta p/\rho) + gZ)]}$$

where $\dot{q}$ = flow rate, m^3/s
A_1 = cross-sectional areas at pipe exit, m^2
g = gravity acceleration, 9.81 m/s^2
$\Delta p = p_1 - p_2$ = pressure drop in Venturi tube, m
$Z = z_1 - z_2$ = difference of elevation head, m
C_d = coefficient of discharge (relation among the areas at throat and exit)

*14.21 Leaks from process piping and oil or gas transmission lines can not only cause pollution but can be extremely dangerous for combustible products if there is a source of ignition. Based on the equation for the volumetric flow rate given in problem 14.20, show that the mass flow rate of a fluid flowing in a pipeline can be measured by the meter using the equation

$$\dot{m} = C_d A \sqrt{2\rho\ \Delta p}$$

where Δp is in Pa
ρ is in kg/m^3
C_d is the orifice coefficient (accounts for irreversibility)
$\dot{m}$ is in kg/s
A = area of pipe, m^2

Let $C_d = 0.65$, and calculate the mass flow rate of CH_4 at 30°C and 307 kPa from a leak with a diameter of 0.4 cm.

The pressure outside the pipe is 99 kPa.